

## ON THE IMPORTANCE OF AFFECTIVE DIMENSIONS OF MATHEMATICS EDUCATION

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**Abstract.** In one of his latest articles, Fortus (2014), points out that “when one considers the centrality of affect to teaching and learning and the broad range of topics that are related to affect, it is concerning that it has received relatively so little attention” (Fortus 2014, p. 821). In order to support his position, he provides an overview of the research on affect in science education that has been published in several journals (JRST, SciEd, and IJSE) between 2001 and 2011. The author also hypothesizes why affect has been under-attended to by the science education research community so far. And the conclusion he arrives at is that affect remains in the shadow of researchers’ attention partly due to the existing “international trend towards standardization of schooling and high-stakes testing” (p. 822). The main purpose of this article is to emphasize that affect does play an important role also in learning mathematics and for this reason it should be considered as one of the core dimensions of mathematics education. The first part of this article provides examples of two phenomena: math anxiety and the underachievement syndrome in learning mathematics, where affective determinants are unquestionable. Subsequently, we shift the focus from these particular issues to the general description of what affect is, what meaningful concepts it contributes in the field of research on mathematics education, and how the research community can benefit from the approach it promotes. Finally, we present some new directions for researchers and teachers that may result in an increase of the quality and efficiency of both teaching and learning mathematics.

**Keywords:** affect in mathematics education, math anxiety, underachievement syndrome.

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### 1. Significant findings on neurological roots of math anxiety

A considerable number of papers have already dealt with the math anxiety problem. Our intention here is neither to provide an overview of the existing research reports, nor to analyze the work that has already been

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done. We focus here on presenting some recent findings that literally show affective underpinnings of mathematics education.

Math anxiety (MA) phenomenon is commonly described in terms of “feeling of tension, apprehension, or fear that interferes with math performance” (Ashcraft 2002, p. 181). Tobias and Weissbrod (1980), defined math anxiety even as “the panic, helplessness, paralysis and mental disorganization that arises among some people when they are required to solve a mathematics problem” (p. 65). Stemming from unpleasant or poor experiences in mathematics, math anxiety is supported by the socially shaped portrait of mathematics as being difficult, detached from reality and, in some way, an esoteric field of knowledge. Timed tests and fear of embarrassment in front of the classroom are important factors that cause or maintain MA. Typically, anxious students experience the fear that they will not be able to deal with mathematical problems or that failure, which they predict from before they start doing mathematics, will reveal their misunderstanding. Negative emotional responses to math stimuli result in a decrease of students’ self-confidence which, in turn, leads to passive behavior. Anxiety keeps students moving in a vicious cycle of disaffection, avoidance and a decrease of understanding the subject matter. Unfortunately, MA is not limited only to school settings. Math avoidance, inevitably resulting in less competency, prevents learners from applying for admission to sought-after fields of study and facilitating better job opportunities.

Although most of the well-known studies have focused on the behavioral aspects of math anxiety, recent findings (Young, Wu, Menon 2012), provide neurobiological evidence for this phenomenon. A sample of 46 second and third graders were given a neuropsychological assessment and two runs in the fMRI (functional magnetic resonance imaging) scanner. During imaging procedures, researchers found some significant differences between the brains of anxious and non-anxious students. The brains of students with identified MA showed hyperactivity in the right amygdala region playing a key role in nonconscious processing of emotion, as well as in the hippocampus, crucial for storing our memories and connecting them to our emotions. Moreover, the study revealed that MA was associated with reduced activity in those brain regions (posterior parietal and dorsolateral prefrontal cortex) which support working memory and numerical processing. When our working memory is overloaded, for example by the flood of information, or by fear or anxiety as well, we are far less able to retrieve and use the information given to be held simultaneously in the mind. These findings are more than important as they show explicitly that math anxiety

is not just a mere excuse that some students use in order to justify their reluctance to engage in mathematical activities. It is now evident that the MA phenomenon occurs in terms of neurological responsiveness of a human's body.

Independently, a group of researchers from the University of Chicago, also interested in the biological underpinnings of MA, have investigated brain activation related to anticipation of doing math and math performance as well. The study involved 14 students with high level of math anxiety and 14 low math-anxious subjects. All participants were identified in a separate prescreening session by using the Short Math Anxiety Rating-Scale (SMARS). Subsequently, while students were solving some word and math tasks, their brain neural activity was being measured by functional magnetic resonance imaging. Surprisingly, this study resulted in some unforeseeable findings. One of the observations (Lyons, Beilock 2012a, 2012b), was that the students' brain activity was induced by the anticipation of having to do math, not actually doing math itself. The second, even more significant discovery, was that the anticipation of doing math activated these brain regions which are responsible for visceral threat detection, including physical pain. Researchers pointed out that "this relation was not seen during math performance, suggesting that it is not that math itself hurts; rather, the anticipation of math is painful" (Lyons, Beilock 2012b). This finding suggests that the experience of pain depends on the psychological interpretation one attributes to his anticipated mathematical activity, rather than the task he is actually dealing with. However, in light of the above results, it is not surprising that students with high level of math anxiety tend to avoid mathematics, either inside or outside the classroom.

Lyons and Beilock (2012a), summarize their findings with a practical pointer for teachers: "(...) **educational interventions emphasizing control of negative emotional responses to math stimuli (rather than merely additional math training) will be most effective** in revealing a population of mathematically competent individuals, who might otherwise go undiscovered" (p. 2102).

Being aware of the fact that students are overwhelmed by fear before they start solving a mathematical task, teachers may try to shorten the anticipation phase and not let students go through the anxiety. This can be achieved by immediate shifting students' attention directly to mathematics. According to the study results, when one begins doing what he is afraid of, the neural activity of one's brain changes in a way that reduces unease and anxiety. What is more, letting students express their emotional state, for

example, in a ten-minute writing activity before the test they expect, can ease students' working memory and improve test performance (Park, Ramirez, Beilock 2014; Beilock, Willingham 2014).

These remarks delineate brand new directions for teacher educators and mathematics teachers. Beilock appeals for educating teachers about math anxiety. They need to know what causes MA, how to identify highly anxious students, how to prevent learners from math anxiety and in the case of students already suffering from anxiety, how to help them to reduce it. Although it is somewhat counter-intuitive, math anxiety also affects both prospective and in-service teachers. The problem, especially when apparent among elementary school teachers, may influence even young children entering school (McAnallen 2010).

## **2. Underachievement Syndrome as the urgent matter for mathematics educators**

In recent years, educators and teachers have put a lot of emphasis on students with special educational needs. According to the Warnock Report (1978), which raised the problem for the first time, there are many children who need professional support to meet the requirements of the core curriculum. As a UNESCO member, Poland has adapted its educational law to the recommendations of the Salamanca Conference (1994). The implementation of the principle of inclusion concerns mostly students detected with some developmental deficits, the mentally or physically handicapped and also the gifted ones. Yet still, those who – for various reasons – underachieve remain in the shadows of public attention.

One of the possible reasons may be that identifying gifted pupils poses many difficulties. Research results reveal (Tokarz, Słabosz 2001; Cieślukowska, Limont 2010), for instance, teachers' tendency to overestimate the ability of diligent and disciplined children. On the other hand, teachers diminish the intellectual and creative potential of passive, unsystematic, disobedient students who contest and resist their efforts. This is all the more astonishing as this was already discussed a few decades ago by Wasyluk-Kuś (1971), who noticed that gifted children may tend to adopt attitudes that would never be accepted by the school system like, for instance: an unwillingness to learn, a dismissive attitude towards school duties or teachers and a lack of persistence. The more current research of Rimm (1994, 1995) and Dyrda (2000, 2007, 2012) shows that there are many undiscovered talents among such students. This is a very serious problem

because if not discovered and developed, talent often disappears (Dyrda 2007). The lack of competence to identify gifted students and the poor individualization of teaching that would enable the early detection of capable students' deficits, lead in particular to the development of the Underachievement Syndrome.

According to some dictionary definitions, the underachiever is:  
a student who performs *less* well in school *than would be expected on the basis of abilities indicated by intelligence and aptitude tests*, etc.

or

a person that *performs below expectations* (*Dictionary.com Unabridged*)

but also

someone (such as a student or athlete) who does not perform as well *or work as hard as he or she can* (*Merriam-Webster.com*)

Rimm (1997), defines underachievement as:

a discrepancy between a child's school performance and some index of the child's ability. If children are not working to their ability in school, they are underachieving (p. 18).

Understated student achievement may be temporary or chronic. In most cases, circumstantial decrease in performance transforms over time into a chronic one (Dyrda 2007). If we take into account the range of Underachievement Syndrome, we can distinguish two types of the syndrome: local – related to one or two school subjects (subject specific underachievement) and global – referred to all school subjects (general underachievement).

In related literature, the causes of the syndrome are usually categorized into three groups (Dyrda 2000, 2007; Rimm 1994, 1995):

1. Student dependent – reasons inherent in the learner; mental and somatic characteristics of the student: level of intellectual and emotional dysfunctions, physical defects, illnesses, but also affective ones: *lack of internal motivation, fear of failure and low self-esteem*, which are considered to be the most decisive factors.

2. Environmental – social, economic and cultural conditions of the family, emotional environment within the family, parenting style, impact of group of peers.

3. School dependent – including poor organization of lessons, the lack of individualization of education, lack of additional classes, incompetent compensatory. Another important problem here is classifying students with the underdog stigma, which acts as a self-fulfilling prophecy (Golem effect). Stigmatized in such a way, students easily fall into apathy and lose motivation. Designated by the authority of the teacher to the group of underdogs,

they also write themselves off and start holding the belief that they will never be good at, for example, mathematics.

Many research reports allow us to outline some attributes and behavior typical for underachievers (Dyrda 2000; Rimm 1994, 1995). Despite of the low-level test outcomes and difficulties in completing the tasks, these students formulate non-trivial questions and hypotheses. Although they put minimal effort into their work in the classroom, they often have broad extra-curricular interests. Behaviourally, they are described in terms of selective focusing attention, the tendency to withdraw into oneself (or to the contrary: offensive domination), hyperactivity (in extreme cases: including ADHD symptoms) and negative attitude to school duties. Most underachievers lack motivation for learning and they are bored with school. They set themselves too low or too ambitious goals.

Rimm (1994), developed the Underachievement Syndrome therapy. Her tri-focus model involving cognitive and emotional components, requires close cooperation between the student, his parents and the teacher. Rimm proposed a therapy consisting of six steps. The first step is the diagnosis comprising formal (tests, inventories, analysis of documents) and informal (conversations, interviews, observations) observing manifestations of the Syndrome. The next steps are: communication, modification of parents' and student's expectations (student status becomes changed in the social – emotional structure of the class), remedial classes filling in the gaps of student's school knowledge and skills (including learning skills) and finally, modifying student's behavior. Some research reports (Reis, McCoach 2002; Del Siegle, McCoach 2005), suggest that successful learners differ from underachievers in the beliefs they have and the way they deal with emotions experienced in relation to anticipating or doing mathematics. As beliefs and emotions belong to the affect domain, it seems important to bring attention to this field of research and explore its results.

### **3. Affective dimensions of mathematics education**

Contradictory to commonly held view of mathematics as a solely intellectual field of human activity, Schoenfeld (1983) acknowledges that “purely cognitive behavior is extremely rare, and that what is often taken for pure cognition, is actually shaped – if not distorted – by a variety of factors” (Schoenfeld 1983, p. 330). In recent decades many researchers have become convinced that affect plays a key role in mathematics education. As stated by McLeod (1992), “if research on learning and instruction is to maximize

its impact on students and teachers, affective issues need to occupy a more central position in the minds of researchers” (p. 575). Also Op’t Eynde, De Corte and Verschaffel (1999), note that “referring only to cognitive and metacognitive factors does not capture the heart of learning” (p. 97).

One of the most frequently cited definition of affect comes from Goldin (1999), to whom affect is “a *system of representation*, encoding information about the external physical and social environment, mathematics, cognitive and affective configurations of the individual, cognitive and affective configurations of others” (p. 37). Similarly to cognition, affect also has its own structure. Below we present a tetrahedral model of affective structure (DeBellis, Goldin 2006, p. 135), encompassing emotions, attitudes, beliefs and values. As this paper is meant to be only a brief exposition of the affective determinants influencing mathematics education, it is impossible here to give a comprehensive overview of the existing literature regarding all related concepts. At the risk of oversimplification, we provide a sketchy description of each affective component, and we encourage the *interested reader to find* more details in further reading.

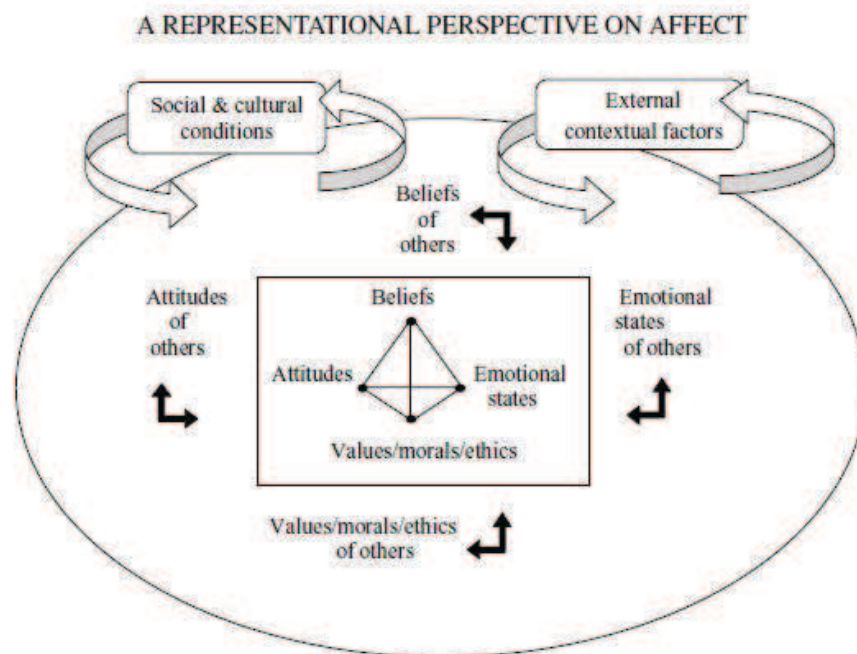


Fig. 1. A tetrahedral model describing domains of affect

Source: DeBellis, Goldin 2006, p. 135.

*Emotions* describe the “rapidly-changing states of feeling experienced consciously, or occurring preconsciously or unconsciously during mathematical (or other) activity” (DeBellis, Goldin 2006, p. 135). They result in physiological and psychological changes that influence one’s thought and behavior. According to McLeod (1992, 1999), it is often the interruption of plans or planned behavior that gives rise to the emotional response implying most of the affective factors. The meaning one attributes to his emotional reactions, however, depends on his knowledge and the set of beliefs he holds.

By *attitudes*, people usually mean some predispositions to certain kinds of behavior, whereas researchers describe them as orientations or predispositions toward certain sets of feelings. Recurring experiences of the same type may result in less intensive and more automated emotional reactions. For McLeod (1992), attitudes lay somewhere in-between the “hot” but flexible emotions and “cold” beliefs.

*Beliefs* are rather stable ways of thinking in which a person roughly regards something to be true. They are formed as a reaction to one’s experiences. They give meaning to what happens in one’s life, prevent people from pain and suffering when they encounter some adversities, and maintain the state of psychological equilibrium. Snow, Corno and Jackson (1996), assume that “human beings in general show tendencies to form and hold beliefs that serve their own needs, desires and goals; these beliefs serve ego-enhancement, self-protective, and personal and social control purposes and cause biases in perception and judgment in social situations as a result” (p. 292). Beliefs do not exist in isolation, rather they occur in clusters. It is possible for one person to hold two contradictory beliefs without any internal conflict, if they are held in separated clusters. People may change beliefs perceived as incompatible (Op’t Eynde, De Corte, Verschaffel 2002). Although beliefs are often highly stable, Moscucci (2007), points out that it is easier to “break off” a rigid element of the structure than the elastic one. Though it seems counter-intuitive, they may be more vulnerable to change than fickle emotions.

*Values* related to deep personal truths serve as a motivation for both long-term choices and shorter-term priorities. This component of affect has not gained sufficient mathematics educators’ attention yet.

Pieronkiewicz (2015), refers to the transgressive concept of man proposed by Koziol (1987, 1997), and appeals for focusing on *the process of change in affect* itself. By the term of *affective transgression*, she means an intentional process of overcoming personal affective barriers that pre-



clude one's mathematical growth and development. The process is a psychological, individual and constructive transgression towards oneself.

Affective transgression might occur if and only if a person has insight into emotions he experiences, is aware of the belief systems he holds and, last but not least, has the will to make changes, believing they are good and possible. Changing one's affect requires improving one's meta-affective competencies. *Meta-affect*, defined as "affect about affect, affect about and within cognition about affect, and the individual's monitoring of affect through cognition (thinking about the direction of one's feelings) and/or further affect" (DeBellis, Goldin 2006, p. 136), is recently considered to be the most important component of affect.

#### 4. Final remarks

To successfully address educational needs of young people today, teachers need to become effective facilitators of learning processes instead of being only mere transmitters of facts and skills applicators. In case of anxious and underachieving students who lay at the heart of our considerations here, researchers recommend providing some therapeutic interventions:

"Seemingly 'small' social-psychological interventions in education – that is, brief exercises that target students' thoughts, feelings, and beliefs in and about school – can lead to large gains in student achievement and sharply reduce achievement gaps even months and years later. (...) By understanding psychological interventions as powerful but context-dependent tools, educational researchers will be better equipped to take them to scale" (Yeager, Walton 2011, p. 267).

To be prepared for this, teachers have to receive some instruction first. Exploring the affect domain, one will find ideas that correlate with recent findings. Lyons and Beilock (2012a), suggest teaching students to control their emotions prior to doing mathematics. The same idea can be found in Moscucci (2007). DeBellis and Goldin (2006), point out that "the development of powerful affective and meta-affective structures, (...) may turn out to be keys that unlock mathematical power in learners" (p. 145). From their vantage point, however, the most important goal in mathematics education is not to "eliminate frustration, remove fear and anxiety, or make mathematical activity consistently easy and fun" (p. 137). They find value in developing students' meta-affective competencies which transform every emotional feeling and difficulty into a productive experience supporting both learning

and accomplishment. Being equipped with such a tool, students are able not only to improve their performance in the classroom, but also become better prepared for future life challenges.

Mathematics education seen through the lenses of affect, not only provides an opportunity for intellectual growth, but also gives a chance to work on one's emotional intelligence and personal development.

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