Abstract. Three-dimensional vibrations of economic vectors always take place around the hypothetical line of the economic equilibrium. The geometrical observation of the kinematics of the course of stock exchange occurrences originated the so called technical analysis, which among other activities seeks new rules in order to be able, with their help, to appoint occurrences of so called turning points on capital markets, or to fix time horizons of appearing trends, etc. In the sequence of several dozen recent years the methods using two-dimensional spirals (price and time) have experienced a few interesting alterations. It seems that the research of capital helices should be continued, since the very process of whirling of vectors will last as long as the ones participating in the market will be undertaking the decisions in terms of the price and quantity, concerning the purchase (or sale) of vectors.

Keywords: golden ratio, vortex movement, logarithmic helix, modified cobweb model.

JEL Classification: F10.

1. Introduction

The paper presents a description of the vortex (chaotic) structure of financial markets. In the literature there are many trials in which attempts have been made to discern some kind of regularity in the chaotic movements of the market. One of the links connecting chaos with market trajectories is the number of the golden ratio.

2. Structure of the vortex

In the reality around us there are many forms that are characterized by their vortex kinematics. Take, for example air, and the fact that 1 cm$^3$ of which contains 20 trillion particles ($20 \times 10^{18}$). Air molecules are in constant vortex movement, and their velocities are about 1600 km/h. Each molecule collides with the others approximately 5 billion times per second. Similar
turbulence of molecules can be observed in various liquids and in other gases. Physicists have known for a long time about spinning electrons. It is known that electrons rotate around their axes, and have an electrical charge, and the spin generates local magnetic field which, depending on the direction of rotation (closely spaced electrons), can give the effect of attraction and repulsion. As it turns out, the atomic nuclei in a great simplification can be regarded as rotating, electrically charged particles. So, at the micro level, we can say that all our surroundings spin around us. Whereas at the macro level the spinning movement is shared by all planets in our Solar System, also the newly discovered galaxies spin around. In the world of medicine, for example, it has been quite recently discovered that the blood in the atria and chambers of the heart spins too. Streams flowing from the veins do not collide with each other, but form large eddies which are then divided into streams of blood and pumped into arteries. We also know that the spiral structure of DNA (deoxyribonucleic acid) hides the greatest amount of genetic information. The mysterious vortex kinematics of different forms stabilizes their dynamics. It seems that the spin is the guarantor of these forms to pursue the so-called equilibrium of system dynamics. Known from antiquity, famous Heraclitus’ *pantha rei* means literally “everything flows”, everything changes.

3. Market structure of the vortex

In the world of economics, size of demand and supply are constantly changing, and is they that just set into motion vectors in the economic wealth, which compose of: price, quantity, time. It turns out (according to a very general statement of Heraclitus) that the economics of three-dimensional systems\(^1\) dominates the vortex kinematics. Although it is widely known that the cause for this kind of movement lies in the value of the already mentioned supply and demand forces that affect the size of the components of three-dimensional vectors, but certainly all factors that affect the sizes of these forces, causing the three-dimensional vortex movement in the economy, are unknown. The whirling movement is inevitably connected with helices (irregular three-dimensional spirals). There has been a tendency to mention spirals in economy using common conversational phrases such as spirals of: price, inflation, remuneration, mining, unemployment, poverty,

\(^1\) \(R^3 = P \times Q \times T\) (Price, Quantity, Time), \(R^3 = I \times U \times T\) (Inflation, Unemployment, Time).
etc. Typically, a description of economic phenomena made with the use of a spiral is associated by the participants of the widely understood market as - bad. And it is related to the fact that the geometric curve, which a spiral is, is identified with a spiral developing in time (immediately increasing its size). Rapid development of spiral dynamics is the effect of kinematic destabilizing of the vectors whose movements observed in the three-dimensional space reveal their true vibratory nature. Also, the process of stabilizing the kinematic contributory value of the vectors, carried out as described above, with the only difference being in three-dimensional space the helices generated by the phenomenon literally coil up. Three-dimensional vibration vectors always take place around a hypothetical line of balance (in this case, the economic balance.) Geometric observations of the kinematic of the economic phenomena development gave rise to the so-called technical analysis which is looking for more and more new rules so that with their help one will be able to set an instance of the so-called turning points in capital markets. In recent decades the method using two-dimensional spirals (price and time) lived to see a few interesting modifications, although many have been recently rejected and not considered to be an effective method of forecasting. However, the search for new methods is still ongoing. One of these new research methods is, for example ermanometry, the basic premise of which is the assertion that the time for capital markets has a multidimensional structure. The vast majority of the so-called analytical techniques at the same time keep track of two kinds of movements, i.e. price movements, vertical (peaks and declines) and horizontal (time factors). In ermanometry, it is time that is the dominant factor for designating price movements in financial markets. It is known that at any time in the market there are many active price formations. In a typical analysis, a cycle of between ten and twenty weeks may be declining, while the one between ten and twenty months presents a growing tendency. In such situations, analysts usually look for points where the majority of short- and medium-term cycles are synchronized together within a short period of time, i.e. when the peaks and declines of both cycles overlap. In ermanometry the points that are sought are the ones in which several development formations (schemas) intersect. So called classical analysis adopts the assumption that the market is growing only in the direction of the future – time². New three-dimensional vortex begin where the old ends. Ermanome-

² Clearly, however, one can recognize the fact that time (as such), does not have a fixed direction.
try assumes that market time is structured spherically and multi-dimensionally, it allows, therefore, the existence of multi-dimensional vortex trajectories – helices.

The emergence of the multidimensional helices in ermanometry is a natural phenomenon. Practically, we can say that the process of spinning of vectors in the economic cubes \( R^3_+ = P \times Q \times T \) continues uninterrupted, as evidenced by the orthogonal projections of the helices in three mutually perpendicular planes (see Fig. 1). At the two planes are the orthogonal projections typical zig-zags depicting increases and decreases. These are the two-dimensional zig-zags that were for many years the subject of studies of researchers who wanted to describe the technical mechanism of the functioning of the stock exchange. One of them was Ralph Nelson Elliott³. He created the basis for a spectacular analysis of flat stock market zig-zags

³ An American accountant who lived in the period 1871-1948 first noticed the numbers and the Fibonacci golden ratio and its reversal in two-dimensional stock market charts. In 1939 he published in the Financial World the assumptions of his theory of waves. This theory (with some modern additions) is used to this day by many analysts.
known today as the Elliott Wave Theory. The mathematical basis for this theory is Fibonacci recursive and golden ratio number of $\chi = 0.618...$, or its inverse $\phi = 1.618...$. The main assumptions of the theory are that, firstly, in the stock market there are temporary formations, in which to the number of zig-zags corresponds to Fibonacci numbers, and secondly that the range of stock (increases and decreases) remains the golden ratio (see Fig. 2).

Fig. 2. Fibonacci numbers and golden ratio of market formations

Source: own calculations based on (Frost, Prechter, 1995).

Elliott’s wave theory fully verifies the fact that rotation of shares (indices) occurs in three-dimensional space. At intervals, during which three-dimensional stock helices take shape similar to logarithmic trajectory, the proportion of flat zig-zags analysis is based on the golden number (or its inverse). It is a natural consequence of the orthogonal transmission (projection) number of the golden ratio that appears in the $R^3$, $R^2$. Another very interesting way of making use of Elliott’s approach is a combination of price and time analysis (known today as a cost-time method). Points that result from the use of this approach – the days of the retreat is usually marked as $DD$, are characterized by a high probability of reversal. The condition for the creation of a $DD$ point is to obtain the same estimates using both the analyses of time and price (see Fig. 3).
Another method derived from Elliott’s concept, combining both time and price analysis is the so-called Fibonacci arcs. In order to draw Fibonacci arcs one must connect with the trend line two points of the price cycle, one of which is a local minimum (point A) and the second local maximum price (point B). Then, taking as a basis for the second of the extreme points three arcs are to be drawn in such a way that they intersect the trend lines at 38.2, 50.0 and 61.8% of the length of this line. The location of so called support and resistance levels occurs when prices reach the level of arcs. Intersections of arcs by a rate line confirm the change of the previous trend of a higher order (see Fig. 4). The following illustration shows how to use and interpret these Fibonacci arcs in a geometric way. By drawing the pulse wave as the first step (trend from point A to point B), as a consequence we obtain three Fibonacci arcs. After a decline in asset prices from point B to C, we can observe an adjustment trend to the point D. You will notice that the price reflection stops at one of the Fibonacci arcs. In the middle arc there can occur short-term increases or decreases in share prices of a given asset. Used (for creating Fibonacci arcs) percentage proportions are limit values of quotients of Fibonacci numbers, i.e. if we denote $f_n$ is the Fibonacci number of recursion thus $\lim f_n/f_{n+1} = 0.618…$ and in turn $\lim f_n/f_{n+2} = 0.382… = \chi / \phi$. 

Fig. 3. Determination of a target day based on the distance between the so-called turning points A and B

Source: own calculations based on (Frost, Prechter, 1995).
Another phenomenon equally spectacularly used by time and price analyzing are the so-called Fibonacci fans. The first step is to determine the trend line between two adjacent pricing extremes. Then, through the second of the created points a vertical line should be drawn. Then, from the first point three trend lines are to be drawn so that they cross the vertical line at Fibonacci percentage levels, i.e. 38.2, 50 and 61.8% (that was drawn earlier). The resulting rays constitute consecutive levels of so-called support or resistance. There are many examples of the use of Elliott’s ideas in technical analysis. One of these was described by Russell in Dow Theory Letters from 21.11.1973, in which he represents several consecutive time periods in which the various turning points in the U.S. stock market are separated by time units belonging to the set of Fibonacci numbers. Other interesting examples can easily be found in the abundant literature on the subject.

Similar to Elliott’s faith in the stock market governed by a simple law built on the natural numbers (Fibonacci recursive) was shared by Gann⁴.

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⁴ William Delbert Gann was born on 6 June 1878 in Luflin in Texas. In 1903, after several years of working in a brokerage company in Texarkana, the investor decided to move to New York where he opened his own brokerage firm. In 1919, Gann focused on investing on his own account. He began publishing daily Demand and Supply Letters which analyzed and predicted changes in commodity prices. Five years later he published his first book, The Truth of the Stock Tape, which concluded, inter alia, its annual predictions and
Currently considered one of the greatest creators of the theory of how to play the stock market. In the 1930s Gann made his considerable fortune thanks to the discovered and skillfully utilized stock market regularities. The theoretical foundations of Gann’s “theory” seem very enigmatic and mysterious. Plummer suggests that Gann himself did not quite know why the methods he had developed worked in practice so aptly (similar suggestions also concern Elliott). Gann was convinced of the fact that the universe which surrounds us oscillates and vibrates. He argued that equities behave like atoms and electrons, which still vibrate in their rhythm. Later he discovered that the events that cause the movement of prices are in a constant state of vibration, which allowed him to create the concept of a vibrating universe. As he would say: *Vibration is fundamental; nothing is exempt from this law; it is universal, therefore applicable to every class of phenomena on the globe.* Until recently it was known that his intuitions were largely true. It turns out that, in time of simultaneous observation (changing in time), stock exchange volumes and turnover before our eyes appear three-dimensional vortex trajectories, confirming the assumptions of Gann, Dow\(^5\), Elliott et al. as to the fact of permanent state of vibration of the stock exchange market (see Fig. 5). Vibrations of shares (indices) exchange are twofold. They vibrate, so the shares (indices) only, being in the three-dimensional cube \( R^3 = P \times Q \times T \), set (within certain time periods) the rotating surfaces of the second degree that do not have collinear axes of rotation. These rotation axes are always parallel to the axis of the vector of time in which the bevelled surface is formed by the rotating vortex trajectory. After determining by stock exchange helices a number of rotating surfaces it is easy to see that they too (rotating surfaces) spin in \( R^3 = P \times Q \times T \). The very process of spinning of rotating surfaces resembles with its character, known from the physics, a precession motion. Intuitively, a polyline, which was formed with the axis of rotation of the second degree surface, can be clearly

\(^{5}\) Charles H. Dow (1851-1902), journalist, founder and first editor of the *The Wall Street Journal* and co-founder of Dow Jones and Company. Dow presented his theory in the pages of magazines headed by himself in a series of articles, but not specifying its name, under which is known today. For the first time the name of Dow theory appeared in the work of S.A. Nelson *The ABC of Stock Speculation*, published in 1903.
identified with the line of economic equilibrium, around which literal rotations of stock exchange helices are made. The above described facts are generally very little known even today, and certainly they were not familiar to the creators of the spectacular stock market theories mentioned in the article. Gann, for example (as we know only today, after a careful examination of his stock market biography), intuitively used the two concepts: spiral – depicting price movements, and wheel – describing the passing and oscillations. Using the logarithmic spiral (although not called by him in this way), marked potential turning points in share prices. Gann believed for example that the last lowest price is important from the psychological point of view for the development of subsequent price levels. Intuitive use of coils proved to be very accurate, (this fact has been known only recently (see Fig. 5)). Spirals in Gann and Elliott concepts are flat of the effects of orthogonal helices projections, which generates the stock market (see Fig. 1).

It is also interesting that the lowest price and the logarithmic spiral are reflected in the so-called Gann’s Price Square. Gann’s Square is a method of determining the future of resistance or support. It involves measuring some time units from the historically lowest level of asset prices. The starting price is always placed in the middle of the square, then, clockwise, systematically increasing by a constant volume (price change) prices are introduced. Numbers that are placed in the arms of the cross formed of horizontal and vertical lines intersecting at the centre of the square, are the most likely areas of price retention in the future. Some modification of this method is the use on the graph of the cardinal square skewed lines (diagonals) designating potential turning points. Unfortunately, Gann did not specify which numbers of the many created at the intersections are really significant, so to conclude, one must say that they are all potentially important. Seemingly disordered sequence of numbers resulting from the price square can be divided into four sets of natural numbers: squares of natural numbers (4, 9, 16, 25), (2, 4, 80), (1, 3, 9) and of course the Fibonacci numbers (1, 2, 3, 5, 8). Gann focused on the so-called rule of three in accordance with which a potential turning point is indicated by observing the three phases of bull or bear market in line with the numbers of the price square (e.g. Fibonacci numbers). In Gann’s theory geometric figures also play a large role. A spiral illustrates the movement of prices, a circle describes the transience and oscillation. In his deliberations the investor also used a square and a triangle, constructed a prognostic method nowadays called Gann fans (structurally similar to the aforementioned Fibonacci fans). Using
this method (as well as the Gann angles method), in theory, allows to determine the turning points of price fluctuations.

The logarithmic spiral that appears in the stock markets is a result of vibration of the market. It is a natural bond which unites the analysis of stock prices with the analysis of time in which these vibrations occur. A logarithmic spiral is a curve that connects price forecasts with time. It turns out that the spiral wound around the lateral surfaces of revolving solids can connect forecasts of prices and volumes over time. The spiral and hidden in the geometrical structure number \( \chi \) are considered by many to be one of the most important and beautiful mathematical curves and ratios\(^6\). The most amazing thing is that the same spiral shape can be seen in capital markets in times of rapid changes, at the time in which behavioural patterns manifest themselves most strongly and clearly. The shape of the logarithmic spiral reflects a process of endless vibration. It develops from its centre at the rate specified by the golden ratio \( \chi \) – so often appearing in the stock market geometry analyses.

What we know about \( \chi \) is that it is an irrationality occurring widely in different quotients of lengths obtained from the shape of this unusual curve – the logarithmic spiral (see Fig. 6).

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\(^6\) I will mention only: James I. Bernoulli, who wrote about the spiral: *eadem mutata resurso – I am reborn the same, though changed*, and others: Leonardo Da Vinci, Isaac Newton, Albert Einstein, composer Ludwig van Beethoven, Wolfgang Amadeus Mozart, Béla Bartók, Claude Debussy, Franz Schubert, Erik Satie.
Quotient 1. Lengths of arcs of logarithmic spiral created over a right angle, divided by the length of the arcs formed over the straight angle – which is complementary to the right angle of the dividend, determine in the quotient of both lengths always the golden number of $\chi$.

$$\frac{\text{arc}(xw)}{\text{arc}(xy)} = \frac{\text{arc}(yx)}{\text{arc}(yz)} = \ldots = \frac{\text{arc}(wy)}{\text{arc}(wz)} = \frac{\sqrt{5} - 1}{2} \approx 0.618.$$

Quotient 2. The sums of the lengths of two sections, inclined to each other at right angles, concluded between the focus and the logarithmic spiral, divided by the sums of the lengths of two segments inclined at an angle to each other at a straight angle passing through the focus, are equal to the quotient of the golden number $\chi$ if and only if the length of the segment – which is a divisor of this quotient – contain the shorter one of the ingredients of the dividend.

$$\left| \frac{sx + sy}{sx + sz} \right| = \frac{sx + sw}{sw + sy} \equiv \ldots = \frac{\sqrt{5} - 1}{2} \approx 0.618.$$
**Quotient 3.** Quotients of the lengths of two sections, inclined to each other at a right angle, included between the logarithmic spiral and its focus, are always equal to the golden number of $\chi$, or its inverse $1/\chi = \phi$.

\[
\frac{|sw|}{|sx|} = \frac{|sx|}{|sy|} = \ldots = \frac{|sy|}{|sz|} = \frac{\sqrt{5} - 1}{2} \approx 0.618.
\]

**Quotient 4.** Lengths of each segment passing through the focus and connecting two points of the logarithmic spiral, divided by the length of the arc of the logarithmic spiral, which ends are in line with the ends of the dividend’s segment, designate in the quotient always the golden number of $\chi$.

\[
\frac{|wy|}{|arc(wy)|} = \frac{|xz|}{|arc(xz)|} = \ldots = \frac{\sqrt{5} - 1}{2} \approx 0.618.
\]

Practically to this day there is no clear answer to the question: why shares (indexes) listed in $R^3_+ = P \times Q \times T$ whirl? The general answer to such a basic question, of course, exists and can be summed up in one sentence saying that the stock market vectors with component $(p_i, q_i, t_i) \in R^3_+, i \in N$ and others, are put in a spin by two main forces acting on them: supply and demand. But as we know, a lot of factors have an influence on the size of these forces. So there is no precise answer. Just as there is no clear and precise answer as to why our planet Earth does a double spinning movement. And the very fact of the Earth’s rotation has been known since the publication in 1543 of *De Revolutionibus Orbium Coelestium*\(^7\). It is known that the stock market helices in certain time periods take the shape of spirals expanded in

\[ R^3_+ = P \times Q \times T. \]

We know from geometry that the logarithmic spiral, set by polar coordinates $(r, \varphi)$ is described by the equation:

\[ r(\varphi) = a \exp(\lambda \varphi), \]

where $a, \lambda > 0$, whereas $\lambda = \cot \psi$, $\psi$ is a constant angle, created by any ray starting from a polar of the spiral with a tangent. A spiral described with the above equation can be created (in a certain period of time) as a result of the perpendicular projection of the helix on a plane $0PQ$ trading. It is also clear

\(^7\) In 1543 Copernicus published in Nuremberg *De Revolutionibus Orbium Coelestium* (On the Revolutions of the Heavenly Bodies, six books).
that stock exchange events taking shape of spiral shaped helices (vortices) are irregular and appear in different cycles – time scales (see Table 1).

Table 1. Designations of vortices – the corresponding figures: rotations and Elliot waves

<table>
<thead>
<tr>
<th>Vortex Type</th>
<th>Helix Rotations</th>
<th>Waves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Great Vortex</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Great super-cycle</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Super Vortex</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Super-cycle</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Vortex</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>Cycle</td>
<td>17</td>
<td>34</td>
</tr>
<tr>
<td>Primary Vortex</td>
<td>72</td>
<td></td>
</tr>
<tr>
<td>Main</td>
<td>72</td>
<td>144</td>
</tr>
<tr>
<td>Secondary Vortex</td>
<td>305</td>
<td></td>
</tr>
<tr>
<td>Secondary</td>
<td>305</td>
<td>610</td>
</tr>
<tr>
<td>Daily Vortex</td>
<td>1292</td>
<td></td>
</tr>
<tr>
<td>Daily</td>
<td>1292</td>
<td>2584</td>
</tr>
<tr>
<td>Hourly Vortex</td>
<td>5473</td>
<td></td>
</tr>
<tr>
<td>Hourly</td>
<td>5473</td>
<td>10,946</td>
</tr>
<tr>
<td>Minute Vortex</td>
<td>23,164</td>
<td></td>
</tr>
<tr>
<td>Minute</td>
<td>23,164</td>
<td>46,328</td>
</tr>
</tbody>
</table>

Source: own work.

If we try to watch the stock helix, in different time scales, we will get a very fast and accurate impression that the turbulence of lower levels of time (scale) causes turbulence in the higher scales. In other words, in three-dimensional stock exchange kinematics, micro-vortices (helices) produce vortices in the higher scales. This irregular vortex process is continuous, with the assumption that stock market transactions are also continuously made. Imagine for a moment that to each of the specified quantity of stock market values we unambiguously assign a colour. Observing in the
\[ R^3 = P \times Q \times T \] multicoloured helices quotes (e.g. minute scale), we can conclude that the chaotic vector vibrations (stock exchange particles), move like particles: air (gas), water (liquids with a viscosity comparable to the kinematic viscosity of water equal to 1 cSt measured at 4 °C)\(^8\). Of course, comparing the stock exchange phenomena with hydromechanic or aerodynamic phenomena today seems to be very daring, but the fact of rotation of molecules, shares, stock market (indices) is undeniable (see Fig. 5). When the stock market helices take the shape of a spiral\(^9\), then it seems easier to produce a short-term forecast of the development of economic vectors’ market. The following three equations present three-dimensional vector helices that form a cone of length \(l\) and radius of the base \(s\). The equations are presented in polar coordinates:

\[
S_L(\varphi) = (a e^{i\varphi} \cos \varphi, \ a e^{i\varphi} \sin \varphi, \ a \frac{l}{s})^T,
\]

\[
S_H(\varphi) = (\frac{a}{\varphi} \cos \varphi, \ \frac{a}{\varphi} \sin \varphi, \ a \frac{l}{\varphi s})^T,
\]

\[
S_A(\varphi) = (a \varphi \cos \varphi, \ a \varphi \sin \varphi, \ a \varphi \frac{l}{s})^T.
\]

4. Conclusions

The hypothesis of the whirling state of stock exchange I formulated has been thoroughly empirically verified\(^{10}\). The three-dimensional stock exchange vibrations have been spotted on the American market, considering a period of over a hundred years of research (on the daily index DJIA, NYSE). The immanent whirling state was also confirmed in the analysis made on the qualities of the Warsaw Stock Exchange, quoted from the very beginning of its creation, and other world stock exchanges. The accumulated research of the three-dimensional structure of the economic vibration and its verification led to the creation of the economic theory of vortices.

\(^{8}\) 1 cSt – a unit of kinematic viscosity centistokes 1 [cSt] = 1 [mm\(^2\)/s]. Kinematic viscosity of the liquid is the ratio of dynamic viscosity of the liquid to its density.

\(^{9}\) Logarithmic, hyperbolic, Archimedes’.

\(^{10}\) The development of science is based on the formulation of hypotheses which are next verified experimentally. Bohr atom model, known from physics, was (at the time of publication), a bold hypothesis explaining inconsistent with the classical mechanics properties of hydrogen. The experiments confirmed its validity, but showed some shortcomings. The hypothesis did not explain the reasons for such behaviour of the atom, which has led to the formulation of the theory underlying the rudiments of quantum physics.
This theory, of course, stems directly from the discoveries of: Gann, Dow, Elliott, who closely adhere to the plane $R^2_+ = P \times T$, (or $R^2_+ = Q \times T$). And next it develops in a three-dimensional space $R^3_+ = P \times Q \times T$, confirming the validity of some of the assumptions made by the “big three” of the previously mentioned analysts. Three-dimensional stock market cube and kinematic helices showing the motion of the vectors still hide in their nature a lot of secrets. The economic theory of vortices brings us significantly closer to explaining at least some of these mysteries. Perhaps it will give us an unequivocal answer to the question: are the golden ratio and the Fibonacci numbers, so often found in the stock exchange geometry, only the result of an orthogonal projection of logarithmic helix onto a plane?

**Literature**


**Websites**